MAT1320C

Dr. Fink - practise sheet for the final exam

1. Compute the following limits.

1. $\lim_{x\to 4} \frac{\ln(x)-\ln(4)}{x-4}$
2. $\lim_{x\to 5^+} \frac{x+1}{x-5}$
3. $\lim_{x\to -2} \frac{x+2}{x^3+8}$
4. $\lim_{x\to 0} \sqrt{9+h} - 3h$
5. $\lim_{x\to 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$
6. $\lim_{t\to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t}$
7. $\lim_{x\to\infty}(\sqrt{9x^2+x} - 3x)$
8. $\lim_{t\to\infty} \frac{\sqrt{x+3x^2}}{4x-1}$
9. $\lim_{x\to\infty} \frac{1+x^6}{x^4+1}$
10. $\lim_{x\to -\infty} \frac{2x^2+x-1}{x^2+x-2}$
11. $\lim_{x\to -\infty} \frac{2x^3+x-1}{x^2+x-2}$
12. $\lim_{x\to\infty} \frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$
13. $\lim_{x\to 0} \frac{\ln(x)}{x}$
14. $\lim_{x\to 0} \frac{\sinh(x)-x}{x^3}$
15. $\lim_{x\to\infty} \frac{e^{u/10}}{u^3}$
16. $\lim_{x\to\infty} x^3 e^{-x^2}$
17. $\lim_{x\to\infty}(x - \ln(x))$
18. $\lim_{x\to 0} \frac{e^x-e^{-x}-2x}{x-\sin(x)}$
19. $\lim_{x\to 1^+} x^{1/(1-x)}$
20. $\lim_{x\to\infty} x^{1/x}$
21. $\lim_{x\to 0^+}(4x+1)^{\cot(x)}$

2. Find the first and second derivatives of the following functions.

1. $f(x) = x^3 + 3x + 1$
2. $y(t) = \sqrt{3t^2 + \cos(t)}$
3. $f(z) = e^{z^2}\sqrt{z+3}$
4. $f(x) = \frac{3+x^2}{5x^2+1}$
5. $f(x) = \frac{\sqrt{3x+1}}{6x^2+1}$
6. $f(x) = \sin(3x^2 e^x)$
7. $f(x) = \frac{\sin(3x)}{\cos(3x)}$
8. $f(x) = \frac{\sin(2x)+1}{\cos(4x)}$
9. $f(t) = \sqrt{\frac{3}{t} + 2t}$
10. $f(y) = y^{500} \cdot e^{y^2}$
11. $f(x) = e^{\cos(5x^2)}$
12. $f(t) = \sqrt[4]{\arctan(t^3) \cdot \sin(t)}$

13. $g(x) = \frac{\sin(x)+3x^3}{\arcsin(x)+7\cos(x)}$

14. $u(y) = \ln(y + e^y)$

15. $s(v) = \cos(v^3 + \tan(v))$

16. $p(x) = \sec(x) \cdot e^{x^3+4\cos(x)}$

3. Find the derivate of the following functions:

1. $\int_0^{3x} \frac{u^2-1}{u^2+1}$
2. $\int_1^{x^2} e^{t^2} dt$
3. $\int_3^{\sqrt{x}} \frac{t^2+1}{2t^4+5t^2+1} dt$
4. $\int_{\sqrt{x}}^{2x} \arctan(t) dt$
5. $\int_0^{x^2} \sin(t^3 + 1) dt$
6. $\int_2^{t^2} \sqrt{4x + 7} dx$
7. $\int_{\sin(t)}^{t^2} \frac{x+3}{\cos(x)}$

4. Differentiate the following and find the equations of the tangent lines at the given points.

1. $x^2-4xy+y^2 = 4$, tangent line at $(1, 1)$
2. $\frac{x^2}{x+y} = y^2 + 1$
3. $x^y + y = x$, tangent line at $(1, 0)$
4. $x \cdot e^y = x - y$
5. $xy = \sqrt{x^2 + y^2}$
6. $2x^2+xy-y^2 = 2$, tangent line at $(1, 1)$
7. $\arctan(x^2y) = x + xy^2$

5. Approximate Integration

Set up the Riemann Sums for the following areas. Do the same with the Trapezoidal rule for the given number of steps and Simpson's rule. Recall that the number of approximation values must be even in that case. Double the given value if needed.

1. $f(x) = \frac{2x}{2x^2+1}$ from 1 to 3, take 4 steps.
2. $f(x) = x^2 + \sqrt{1 + 2x}$, $4 \leq x \leq 7$, take 9 steps.
3. $f(x) = \sqrt{sin(x)}$, $-\pi/2 \leq x \leq \pi/2$, take 8 steps.
4. $f(x) = e^{5x}$, $0 \leq x \leq 5$, take 5 steps.
5. $f(x) = \tan(4x)$, $0 \leq x \leq \pi/6$, take 3 steps.

6. Compute the following integrals:

1. $\int_1^9 \sqrt{x}dx$
2. $\int \frac{1+x}{1+x^2}dx$
3. $\int \frac{z^2}{z^3+1}dz$
4. $\int \frac{x}{1+x^4}dx$
5. $\int (x-1)e^{(x-1)^2}dx$
6. $\int x^2\sqrt{2+x}dx$
7. $\int \frac{\sin(2x)}{1+\cos^2(x)}dx$
8. $\int \sec^2(\theta)\cdot\tan^3(\theta)d\theta$
9. $\int \frac{e^u}{(1-e^u)^2}du$
10. $\int \frac{\sin(x)}{1+\cos^2(x)}dx$
11. $\int_0^{\pi/6} \frac{\sin(t)}{\cos^2(t)}dt$
12. $\int \cos^3(\theta)\sin(\theta)d\theta$
13. $\int 5^t\cdot\sin(5^t)dt$
14. $\int \cos(1+5t)dt$
15. $\int_e^{e^4} \frac{dx}{x\cdot\sqrt{\ln(x)}}dx$
16. $\int_0^1 \frac{dx}{(1+\sqrt{x})^4}dx$
17. $\int_0^1 x\cdot e^{-x^2}dx$

7. More integrals.

1. $\int x\cdot\cos(5x)dx$
2. $\int x^2e^{-x}dx$
3. $\int(\ln(x))^2dx$
4. $\int t^4\cdot\ln(t)dt$
5. $\int \ln(\sqrt{x})dx$
6. $\int_0^{2\pi} x^2\cdot\sin(x)dx$
7. $\int_1^5 \frac{M}{e^M}dM$
8. $\int_1^2 x^4\cdot(\ln(x))^2dx$
9. $\int e^{\cos(t)}\cdot\sin(2t)dt$
10. $\int x\cdot\ln(1+x)dx$
11. $\int_e^{e^2} \frac{5(\ln(x))^{1/5}}{x}dx$
12. $\int \frac{x^3}{\sqrt{x^2+4}}dx$
13. $\int \frac{x^3}{\sqrt{x^2-4}}dx$
14. $\int \frac{4+x}{(1+2x)(3-x)}dx$
15. $\int \frac{x^3+1}{x^3-3x^2+2x}dx$
16. $\int \frac{4y^2-7y-12}{y(y+2)(y-3)}dy$
17. $\int \frac{4+x}{(1+2x)(3-x)}dx$
18. $\int \frac{4x}{x^3+x^2+x+1}dx$
19. $\int \frac{x^3+6x-2}{x^4+6x^2}dx$

8. And more integrals.

1. $\int_0^8 \sin(x)dx$
2. $\int_1^9 \sqrt{x}dx$
3. $\int_0^1 (x^e + e^x)dx$
4. $\int_1^{\sqrt{x}} \frac{z^2}{z^4+1}dz$
5. $\int_1^8 x^{-2/3}dx$
6. $\int_0^4 2^s ds$
7. $\int_1^{18} \sqrt{\frac{3}{z}}dz$
8. $\int_0^1 (1+r)^3 dr$
9. $\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}}dx$
10. $\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2}dx$

9.Curve Sketching Sketch the following curves, using the guidelines discussed in class and in the textbook Chapter 4.5.

1. $y = \frac{x}{\sqrt{x^2+1}}$
2. $y = x - 3x^{1/3}$
3. $y = \sin^3(x)$
4. $y = x + \cos(x)$
5. $y = \frac{x^2+5x}{25-x^2}$
6. $y = \frac{x-x^2}{2-3x+x^2}$

10. Optimisation

Exercises from Chapter 4.7: 2-18

11. Newton's method

Use Newton's method with the specified initial approximation x_1 to find x_3, the third approximation to the root of the given equation.

1. $2x^3 - 3x^2 + 2 = 0, \quad x_1 = -1$
2. $\frac{2}{x} - x^2 + 1 = 0, \quad x_1 = 2$
3. $x^7 + 4 = 0, \quad x_1 = -1$